Published by Institute of Physics Publishing for SISSA/ISAS
Received: *August 10, 2003*
Accepted: *September 9, 2003*

Yang-Mills action from open superstring field theory

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Abstract: We calculate the effective action for nonabelian gauge bosons up to quartic order using WZW-like open superstring field theory. After including level zero and level one contributions, we obtain with 75% accuracy the Yang-Mills quartic term. We then prove that the complete effective action reproduces the exact Yang-Mills quartic term by analytically performing a summation over the intermediate massive states.

Keywords: Superstrings and Heterotic Strings, String Field Theory.

JHEP09(2003)022

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Contents

JHEP09(2003)022

1. Introduction

Although the conventional way to compute effective actions in string theory uses two-dimensional sigma model methods, one can also compute the effective action using string field theory methods by integrating out the massive fields. This was shown for the open bosonic string in a recent paper by Coletti, Sigalov and Taylor [1], based on earlier work by Taylor [2]. Since gauge transformations in string field theory mix the massless and matter fields, it is interesting to see how the usual low-energy gauge invariances are recovered after integrating out the massive fields.

In this paper, we generalize the methods of [2, 1] for the superstring using the WZW-like version of open superstring field theory [3, 4]. After including level zero and level one contributions to the effective action, we find that the correct Yang-Mills quartic coupling is reproduced with 75% accuracy. We then show by performing an explicit summation over all intermediate massive states that the exact Yang-Mills quartic coupling is reproduced in both bosonic string field theory and in WZW-like superstring field theory. Since there are several alternative proposals [5]–[7] for open superstring field theory, it would be interesting to know if any of these alternative proposals are able to reproduce these results.

In section 2 of this paper, we compute the purely massless contribution to the effective action, i.e. the contribution before integrating out massive fields. Unlike the bosonic case, there is already a quartic coupling in the WZW-like string field theory action before integrating out massive fields. However, it reproduces the correct Yang-Mills quartic coupling with only 50% accuracy.

In section 3 of this paper, we include the contribution from integrating out level one massive fields. At this level, the correct Yang-Mills quartic coupling is reproduced with 75% accuracy.

Finally, in section 4 of this paper we prove by an explicit summation that the correct Yang-Mills quartic coupling is reproduced after integrating out all massive states. This proof is given both for Witten's bosonic string field theory and for WZW-like superstring field theory. The proof confirms an earlier result [8] that WZW-like superstring field theory reproduces the correct on-shell four-point amplitudes without contact-term divergences.

2. Contribution of massless fields

In this section, we shall compute the purely massless contribution to the effective action. In WZW-like open superstring field theory, the action on a BPS D-brane is [3, 4]

$$S = \frac{1}{2g^2}\left\langle \left(e^{-\Phi}Qe^{\Phi}\right)\left(e^{-\Phi}\eta_0 e^{\Phi}\right) - \int_0^1 dt\left(e^{-t\Phi}\partial_t e^{t\Phi}\right)\left\{\left(e^{-t\Phi}Qe^{t\Phi}\right),\left(e^{-t\Phi}\eta_0 e^{t\Phi}\right)\right\}\right\rangle, \quad (2.1)$$

where Φ is a Neveu-Schwarz string field in the GSO(+) sector at picture zero and ghost-number zero in the large Hilbert space, Q is the BRST charge

$$Q = \oint \frac{dz}{2\pi i}\left[c(T_m + T_{\xi\eta} + T_\phi) + c\partial c b + i\sqrt{\frac{2}{\alpha'}}\eta e^{\phi}\psi_\mu \partial X^\mu - \eta\partial\eta e^{2\phi} b\right], \quad (2.2)$$

and we use the normalization $\langle \xi(z) c\partial c\partial^2 c(w) e^{-2\phi(y)}\rangle = 2$.[1] For practical calculations, it is convenient to use the somewhat simplified form of the action [9]

$$S = \frac{1}{g^2}\sum_{M,N=0}^{\infty}\frac{1}{(M+N+2)!}\binom{M+N}{M}(-1)^N\left\langle (Q\Phi)\,\Phi^M\,(\eta_0\Phi)\,\Phi^N\right\rangle. \quad (2.3)$$

At the massless level, there are two string fields

$$\begin{aligned}\Phi_A &= \int d^dk\, A_\mu(k)\mathcal{V}_A^\mu(k)\,,\\ \Phi_B &= \int d^dk\, B(k)\mathcal{V}_B(k)\,,\end{aligned} \quad (2.4)$$

where

$$\begin{aligned}\mathcal{V}_A^\mu(k) &= c\xi\, e^{-\phi}\psi^\mu e^{ikX}\,,\\ \mathcal{V}_B(k) &= c\partial c\,\xi\partial\xi\, e^{-2\phi}e^{ikX}\end{aligned} \quad (2.5)$$

are the massless vertex operators for the gluon and the auxiliary Nakanishi-Lautrup field. One can easily compute that

$$\begin{aligned}Q\mathcal{V}_A^\mu(k) &= -\alpha' k^2 c\partial c\,\xi\, e^{-\phi}\psi^\mu e^{ikX} + \\ &\quad + \sqrt{2\alpha'}c\left(k\cdot\psi\,\psi^\mu + \frac{i}{\alpha'}\partial X + (:\eta\xi: + \partial\phi)k^\mu\right)e^{ikX} + \eta e^{\phi}\psi^\mu e^{ikX}\end{aligned} \quad (2.6)$$

[1] Since we will only compute the bosonic contribution to the effective action, we can use the $d = 10$ Neveu-Schwarz version of the WZW-like action. However, one could easily compute the fermionic contribution to the effective action using the $d = 4$ supersymmetric version of the action [10] as in [11].

JHEP09(2003)022

and

$$Q\mathcal{V}_B(k) = \sqrt{2\alpha'}c\partial c\,\xi\, k.\psi\, e^{-\phi}e^{ikX} + (-\partial c + 2c(:\eta\xi: + \partial\phi))\, e^{ikX}\,. \tag{2.7}$$

The contribution of these fields to the string field theory action is easily computed to be

$$S = \frac{1}{g^2}\int d^dx\, \mathrm{Tr}\Big[\frac{\alpha'}{2}A_\mu\partial^2 A^\mu - i\sqrt{2\alpha'}B\,\partial^\mu A_\mu + B^2 - \frac{i\sqrt{2\alpha'}}{2}\partial_\mu A_\nu\,[A^\mu, A^\nu] + \\ + \frac{1}{8}A_\mu A_\nu A^\mu A^\nu - \frac{1}{2}A_\mu A_\nu A^\nu A^\mu\Big]\,. \tag{2.8}$$

Note that the cubic coupling of $A_\mu A^\mu B$ vanishes in the superstring just as it does in the bosonic string. In addition, the coupling $A_\mu\, \partial^\mu BB$ also vanishes for the superstring.

After integrating out the auxiliary field B we find

$$S = S_{\mathrm{YM}} - \frac{1}{4g^2}\int d^dx\, \mathrm{Tr}\left[\frac{1}{2}A_\mu A_\nu A^\mu A^\nu + A_\mu A_\nu A^\nu A^\mu\right], \tag{2.9}$$

where

$$S_{\mathrm{YM}} = -\frac{\alpha'}{4g^2}\int d^dx\, \mathrm{Tr}\left(\partial_\mu A_\nu - \partial_\nu A_\mu + \frac{i}{\sqrt{2\alpha'}}\,[A_\mu, A_\nu]\right)^2\,. \tag{2.10}$$

So the quartic terms in the effective action at level zero,

$$S_{\mathrm{zero}} = \frac{1}{g^2}\int d^dx\, \mathrm{Tr}\left[\frac{1}{8}A_\mu A_\nu A^\mu A^\nu - \frac{1}{2}A_\mu A_\nu A^\nu A^\mu\right], \tag{2.11}$$

are off by approximately 50% from the gauge-invariant quartic terms which are

$$S_{\mathrm{exact}} = \frac{1}{g^2}\int d^dx\, \mathrm{Tr}\left[\frac{1}{4}A_\mu A_\nu A^\mu A^\nu - \frac{1}{4}A_\mu A_\nu A^\nu A^\mu\right]. \tag{2.12}$$

3. Contribution of level one massive fields

To compute the contribution of massive fields to the quartic Yang-Mills term, one only needs to consider massive vertex operators $\mathcal{V}_C(k)$ for which the cubic vertex

$$\langle \mathcal{V}_C, Q\mathcal{V}_A * \eta_0\mathcal{V}_A\rangle \tag{3.1}$$

is non-vanishing at zero momentum. Furthermore, we shall only consider massive vertex operators in Siegel gauge, i.e. those vertex operators satisfying $b_0\mathcal{V}_C = 0$.

One can easily show that the only two massive vertex operators at level one which satisfy the above requirements are

$$\mathcal{V}_C^{\mu\nu}(k) = c\,\xi\, e^{-\phi}\psi^\mu\partial X^\nu e^{ikX} \tag{3.2}$$

and

$$\mathcal{V}_D(k) = c\,\partial^2 c\,\xi\,\partial\xi e^{-2\phi}e^{ikX}\,. \tag{3.3}$$

However, the vertex operator $\mathcal{V}_D$ has vanishing quadratic coupling with itself and with $\mathcal{V}_C^{\mu\nu}$ and therefore does not contribute to the quartic Yang-Mills term. So the only contribution in Siegel gauge to the quartic term from level one massive states comes from the state $C_{\mu\nu}(k)$ which couples to $\mathcal{V}_C^{\mu\nu}$.

JHEP09(2003)022

Computing the quadratic and cubic interactions for $\mathcal{V}_C^{\mu\nu}$, one finds

$$S = \frac{1}{g^2}\int d^d x \,\mathrm{Tr}\left[\frac{\alpha'}{4}C_{\mu\nu}C^{\mu\nu} - i\frac{\sqrt{2\alpha'}}{3\sqrt{3}}C^{\mu\nu}\{A_\mu, A_\nu\}\right]. \qquad (3.4)$$

So after integrating out the $C_{\mu\nu}$ field, one finds that the level one massive contribution in Siegel gauge to the quartic term is

	$A_\mu A_\nu A^\mu A^\nu$	$A_\mu A^\mu A_\nu A^\nu$
level 0	0.50	-2.00
level 1	1.09	-1.41
exact	1.00	-1.00

Table 1: Coefficients of the quartic term for the gauge field normalized by a factor $4g^2$.

$$S_{\text{one}} = \frac{1}{g^2}\int d^d x \,\mathrm{Tr}\left[\frac{4}{27}A_\mu A_\nu A^\mu A^\nu + \frac{4}{27}A_\mu A_\nu A^\nu A^\mu\right]. \qquad (3.5)$$

Summing this level one contribution to the level zero contribution of (2.11), one finds

$$S_{\text{zero}} + S_{\text{one}} = \frac{1}{g^2}\int d^d x \,\mathrm{Tr}\left[\frac{59}{216}A_\mu A_\nu A^\mu A^\nu - \frac{19}{54}A_\mu A_\nu A^\nu A^\mu\right], \qquad (3.6)$$

which is off from the correct quartic coupling of (2.12) by approximately 25%.

For comparison, we show in table 1 the numerical values of the coefficients of the quartic term at zeroth and first levels together with their exact values. The results indicate good convergence properties similar to those of the bosonic string [1].

4. Analytic calculation of the exact quartic term

In this section we compute the exact Yang-Mills quartic term in the effective action by analytically performing the sum over all intermediate massive states. This is done first for Witten's bosonic string field theory and then for the WZW-like superstring field theory. Note that we are able to avoid numerical computations like that of [1, 2] because we are only interested in the zero momentum contribution of the gauge fields.

4.1 Quartic term for the bosonic string

To find the massive contribution to the Yang-Mills quartic term at zero momentum, consider the string field

$$\Phi = R + \Phi_A\,, \qquad (4.1)$$

where R is the massive part of the string field and

$$\Phi_A = i\sqrt{\frac{2}{\alpha'}}\,A_\mu c\partial X^\mu \qquad (4.2)$$

is the zero-momentum part of the gauge boson. The contribution of R to Witten's action is

$$S = -\frac{1}{g^2}\left[\frac{1}{2}\langle R, QR\rangle + \langle R, \Phi_A * \Phi_A\rangle + \langle \Phi_A, R * R\rangle + \frac{1}{3}\langle R, R * R\rangle\right]. \qquad (4.3)$$

Shifting

$$R \to R - \frac{b_0}{L_0}\Phi_A * \Phi_A \qquad (4.4)$$

JHEP09(2003)022

to eliminate the terms linear in R, one learns that the massive contribution to the quartic Yang-Mills term is

$$S^{(4)} = \frac{1}{2g^2}\left\langle \frac{b_0}{L_0}\Phi_A * \Phi_A, Q\frac{b_0}{L_0}\Phi_A * \Phi_A\right\rangle . \tag{4.5}$$

This term can be conveniently expressed in the operator formalism as[2]

$$\begin{aligned} S^{(4)} &= \frac{1}{2g^2}\left(\frac{2}{\alpha'}\right)^2 \int d^d x \; \mathrm{Tr}(A_\mu A_\nu A_\rho A_\sigma) \times \\ &\times \left\langle c\partial X^\mu(-\sqrt{3})\, c\partial X^\nu(\sqrt{3})\, U_3 \frac{b_0}{L_0} U_3^\dagger \, c\partial X^\rho\left(\frac{1}{\sqrt{3}}\right)\, c\partial X^\sigma\left(-\frac{1}{\sqrt{3}}\right)\right\rangle \end{aligned} \tag{4.6}$$

where U_r is a representation of the conformal map $f_r(z) = \tan\left(\frac{2}{r}\arctan z\right)$ on the Hilbert space of the string, i.e. $U_r \mathcal{O}(z) U_r^{-1} = f_r \circ \mathcal{O}(z) = (f_r')^d \mathcal{O}(f_r(z))$. (The last equality is true only for primary operators of dimension d.) As discussed in [12, 13], the U_r operators can be written down explicitly as exponentials in the Virasoro operators.

Since the state $c\partial X^\rho\left(1/\sqrt{3}\right)\, c\partial X^\sigma\left(-1/\sqrt{3}\right)|0\rangle$ is a BRST exact state, we can freely commute the propagator b_0/L_0 through the U operators. Then using the identity [13]

$$U_3 U_3^\dagger = U_{8/3}^\dagger U_{8/3} \tag{4.7}$$

we can commute U to the right and $U^\dagger$ to the left with the help of

$$U_{8/3} c\partial X^\sigma\left(\pm\frac{1}{\sqrt{3}}\right) U_{8/3}^{-1} = c\partial X^\sigma(\pm a)\,, \tag{4.8}$$

where

$$a = \tan\frac{\pi}{8} = \sqrt{2} - 1\,. \tag{4.9}$$

Now we can easily evaluate the correlator in (4.6) as

$$\begin{aligned} &\left\langle c\partial X^\mu\left(-\frac{1}{a}\right) c\partial X^\nu\left(\frac{1}{a}\right) \frac{b_0}{L_0} c\partial X^\rho(a)\, c\partial X^\sigma(-a)\right\rangle = \\ &= -\left(\frac{\alpha'}{2}\right)^2 4\int_0^\infty dt\, e^{-t}\left(e^{-2t}a^2 - \frac{1}{a^2}\right)\left(\frac{e^{2t}}{16}\eta^{\mu\nu}\eta^{\rho\sigma} + \frac{\eta^{\mu\rho}\eta^{\nu\sigma}}{\left(\frac{1}{a} + e^{-t}a\right)^4} + \frac{\eta^{\mu\sigma}\eta^{\nu\rho}}{\left(\frac{1}{a} - e^{-t}a\right)^4}\right) \\ &= \left(\frac{\alpha'}{2}\right)^2\left(-\frac{3}{2}\eta^{\mu\nu}\eta^{\rho\sigma} + \frac{1}{2}\eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho}\right), \end{aligned} \tag{4.10}$$

where we used

$$\frac{b_0}{L_0} = \int_0^\infty dt\, b_0\, e^{-tL_0}\,. \tag{4.11}$$

Note that all the terms in the integral are convergent but one, which we defined by analytic continuation to be

$$\int_0^\infty dt\, e^t = -1\,. \tag{4.12}$$

[2] By inserting a complete set of states on the left and right hand sides of b_0/L_0, we can make contact with the calculation of [1], albeit in a different language.

JHEP09(2003)022

This term arises due to the open string tachyon propagating as an intermediate state and, for this state, the exponential representation of the propagator is invalid. So one could either treat the tachyon separately or use (4.12) as a convenient shortcut.

Plugging the result for the correlator (4.10) into (4.6) we find

$$S^{(4)} = -\frac{1}{4g^2}\int d^dx\,\mathrm{Tr}\left(A_\mu A^\mu A_\nu A^\nu - A_\mu A_\nu A^\mu A^\nu\right) \tag{4.13}$$

which is the correct quartic term for the Yang-Mills action

$$S_{eff} = -\frac{\alpha'}{4g^2}\int d^dx\,\mathrm{Tr}\left(\partial_\mu A_\nu - \partial_\nu A_\mu - \frac{i}{\sqrt{2\alpha'}}\left[A_\mu, A_\nu\right]\right)^2 . \tag{4.14}$$

4.2 Quartic term for the superstring

In this section we generalize the analytic calculation of the quartic term to the superstring using the WZW-like action. Substituting the string field

$$\Phi = R + \Phi_A \tag{4.15}$$

into the WZW-like action where R is the massive part of the string field and

$$\Phi_A = A_\mu\, c\,\xi e^{-\phi}\psi^\mu \tag{4.16}$$

is the zero-momentum part of the gauge boson, one obtains

$$S = -\frac{1}{2g^2}\langle\eta_0 R, QR\rangle - \frac{1}{2g^2}\langle\eta_0 R, (Q\Phi_A)*\Phi_A - \Phi_A*(Q\Phi_A)\rangle + \cdots , \tag{4.17}$$

where ... are higher-order terms in R which will not contribute to the quartic Yang-Mills term. Shifting

$$R \to R + \frac{1}{2}\frac{b_0}{L_0}\left(\Phi_A * Q\Phi_A - Q\Phi_A * \Phi_A\right) \tag{4.18}$$

to eliminate the terms linear in R, one learns that the massive contribution to the quartic Yang-Mills term for the superstring is

$$S^{(4)} = -\frac{1}{2g^2}\frac{1}{4}\left\langle Q\frac{b_0}{L_0}(\Phi_A * Q\Phi_A - Q\Phi_A * \Phi_A), \eta_0\frac{b_0}{L_0}(\Phi_A * Q\Phi_A - Q\Phi_A * \Phi_A)\right\rangle . \tag{4.19}$$

Simple evaluation as in the bosonic case gives

$$\begin{aligned}
S^{(4)} &= \frac{1}{4g^2\alpha'}\int d^dx\ \mathrm{Tr}(A_\mu A_\nu A_\rho A_\sigma)\times\\
&\quad\times\Big\langle\left(c\xi e^{-\phi}\psi^\mu(-\sqrt{3})\,c\partial X^\nu(\sqrt{3}) - c\partial X^\mu(-\sqrt{3})\,c\xi e^{-\phi}\psi^\nu(\sqrt{3})\right)\,U_3\frac{b_0}{L_0}U_3^\dagger\times\\
&\qquad\times\left(ce^{-\phi}\psi^\rho\left(\frac{1}{\sqrt{3}}\right)\,c\partial X^\sigma\left(-\frac{1}{\sqrt{3}}\right) + c\partial X^\rho\left(\frac{1}{\sqrt{3}}\right)\,ce^{-\phi}\psi^\sigma\left(-\frac{1}{\sqrt{3}}\right)\right)\Big\rangle\\
&= -\frac{1}{g^2}\int d^dx\ \mathrm{Tr}(A_\mu A_\nu A_\rho A_\sigma)\times\\
&\quad\times\int_0^\infty dt\,e^{-t}\left(e^{-2t}a^2 - \frac{1}{a^2}\right)\left(\frac{\eta^{\mu\rho}\eta^{\nu\sigma}}{\left(\frac{1}{a}+e^{-t}a\right)^4} + \frac{\eta^{\mu\sigma}\eta^{\nu\rho}}{\left(\frac{1}{a}-e^{-t}a\right)^4}\right)\\
&= \frac{1}{4g^2}\int d^dx\,\mathrm{Tr}\left[\frac{1}{2}A_\mu A_\nu A^\mu A^\nu + A_\mu A_\nu A^\nu A^\mu\right],
\end{aligned} \tag{4.20}$$

JHEP09(2003)022

which cancels precisely the unwanted term in (2.9). This cancellation is guaranteed since we are effectively calculating the on-shell string diagram for the scattering of four gauge bosons at zero momentum. But the WZW-like string field theory was proven in [8] to reproduce the correct on-shell four-point amplitudes. So the calculation here can be regarded as an independent check for this proof.

Acknowledgments

We would like to thank E. Coletti, I. Ellwood, I. Sigalov and W. Taylor for useful discussions. The work of M.S. was supported in part by DOE contract #DE-FC02-94ER40818. N.B. would like to thank MIT for its hospitality where this work was initiated, and FAPESP grant 99/12763-0, CNPq grant 300256/94-9 and Pronex grant 66.2002/1998-9 for partial financial support.

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JHEP09(2003)022